Analyzing music with Linear Algebra

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It might not seem so apparent at the surface level, but mathematics and music have always been closely related. In simple words, mathematics is a science that implements logic to explain patterns, shapes, and numbers [1]. It is essentially the building blocks of everything that happens in people's daily lives. Music, as an art form, is defined as the "combination" or "ordering" of sounds to "produce a composition having unity and continuity." [2] These two different studies of art and science relate in that one aids the understanding of the other.

The linkage between the two was discovered long ago by the Greek mathematician and philosopher Pythagoras, who revealed the connection between pitch intervals and mathematical portions [3]. It is interesting that throughout the years, both music and math continued developing, with the latter continuing to support understanding of the foremost. Today using linear Algebra, people can analyze and compare music with one another in a mathematical way [4] that will potentially open up doors for further application of Linear Algebra in music.

Music is composed of many different parts. There are notes on the manuscript assigned to distinct pitches that notify the musicians of which notes are played, rests to indicate the duration of pause or void of notes, and power, which indicates strength and attack on individual notes [3].

To translate music into mathematics and be analyzed, the conventional way of naming the pitches (e.g. C, C#, D, D#, etc.) must be replaced by mathematical notations that would allow further examination of musical characteristics [5]. And Linear Algebra is the obvious choice as vectors and matrices can clearly demonstrate the relationship between each note and create a three-dimensional space for a melody [5]. By three-dimensional, it means that each note can be described in terms of its pitch, duration value, and power, as mentioned above.

How is this done? This is approached by creating matrices known as "Melody Matrices," comprised of Note vectors, Duration Vectors, and Power vectors [5]. Whether it be classical, rock, jazz, and contemporary songs, every piece of music always have a central melody. That melody is a recurring theme throughout the whole piece that could be analyzed using matrices.

To define note vectors, the musical notations in the manuscript for {C, C#, D, D#, E, F, F#, G, etc.} are replaced numbers such as {0,1,2,3,4,5,6,7,8…} as the frequency of these pitches are increasing [5]. It is intuitive that the higher the number for the corresponding pitch, the higher the note is. For a simple example and applying the steps mentioned in [5], a monophonic (single lined melody) is used. In the example melody given below in figure 2, the note vectors being translated for bar 1 (the first bar to the left separated by and vertical line) contain notes {C,A,G,F,F}. As note vectors, these are translated to [12 9 7 5 5].

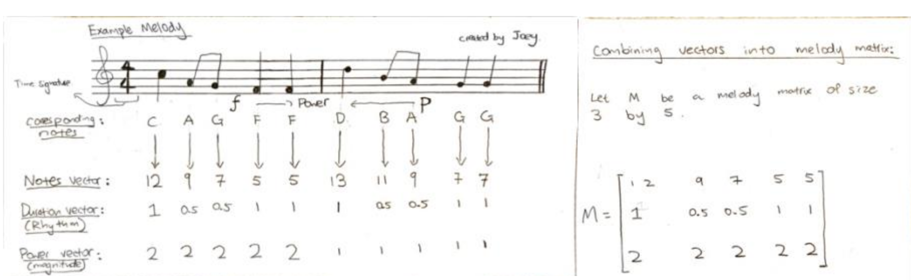

Figure 1: The example melody is created by myself and demonstrates a relationship between the notes, duration and power on the manuscript and numbers used to represent them. On the right, there is an example melody matrix constructed to show first bar of the melody.

Next, the Duration and power vector needs to be defined as well. In the example melody above, the duration vectors (rhythm) depend on the time signature (4/4 meaning four beats per bar). The value of 1 and 0.5 are assigned to each note, with 1 signifying a note taking up one beat and 0.5 meaning a note takes up half of a beat. Adding them together would equal a total of four beats. The power vector, in this case, is assigned as either a value of 1 or 2. The number 2 refers to the forte (loud), and the number 1 relates to the piano (quiet). As demonstrated in figure 2, bar one has universal power vectors of value 2 and bar two has value 1. Putting these vectors together, a melody matrix is obtained [5]. This is shown on the right of figure 2, with "M" being the melody matrix for bar 1.

These melody matrices can allow characteristic analysis of music [5]. For example, by comparing the melody matrix of bar one to that of bar two in figure 2, it would be noticed that the difference between them is subtle. The note vectors are changed up by a value of 2 (or transposed a whole tone up in music theories), with the duration vectors remaining the same. The power vectors are reduced by half because bar two is being played quieter.

Furthermore, the idea of melody matrices can be applied to music compositions. The generative adversarial network (GAN) model is a method that composes music based on bars [6]. It involves the idea of using melody matrices extracted from MIDI (Music Instrument Digital Interfaces) files to generate new melody matrices [6]. As a side note, MIDI files contain a lot of information regarding tempo, time signature, and song structure [7]. Here, the new melody matrices are constructed into new MIDI files through transformations; then, a melody is created using the new MIDI file (a general process can be seen in Figure 3.)

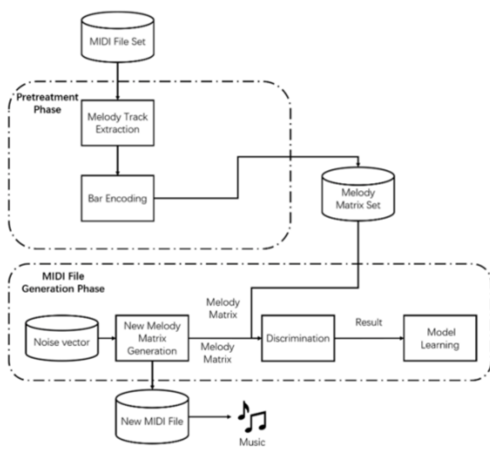

Figure 2 Process of melody generation using GAN. Source: adapted from [6]

The possible applications of linear algebra in music are huge. Using matrices and vectors to simplify and translate musical languages to mathematical interpretations can allow massive numbers of musical pieces be clearly compared and contrasted with each other. It can even help model musical structures and be manipulated and applied to generating an algorithm for music creation [5].

References:

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