Modelling Project – Balloons and the Leaning Tower of Pisa

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Every year, the leaning tower of Pisa is estimated to move at a rate of 1.2 millimeters per year [1]. It is scary to think that this world-famous building might one day lean so much that it falls to its destruction. To make sure this doesn't happen, I have tasked myself to devise a mathematical model that will possibly solve this problem and hopefully not only make sure the leaning tower of Pisa not fall anymore, but completely correct its tilt. This plan, of course, involves attaching helium balloons onto the building and using the lift force of the balloons to lift the tower back into its original position. In this modelling project, I will be mainly trying to estimate how many balloons will be required to make the leaning tower revert back to a complete vertical position.

To begin, a general model is needed to examine the situation. The leaning tower of Pisa has a natural tilt of 5. 5 degrees from its vertical, perpendicular to the ground [2]. This was caused by the tower's foundation which was built on soft, shifting grounds [1] and the weight of the tower pulling it down. Today, parts of the base of the tower are submerged in the ground beneath. For this project, the general model we will be working with would consist of the body, bell tower, and the natural tilt of the tower represented in two Dimensions (as can be seen in figure 1). For further simplification and to make this model a bit more manageable, the rigid ground beneath the tower is replaced with a hypothetical pin support that would allow the tower to rotate and have a moment at point B (figure 2.). Furthermore, this pin support would also disallow the tower to displace horizontally, making the model a lot simpler and allowing us the convenience to not account for the ground holding the tower down.

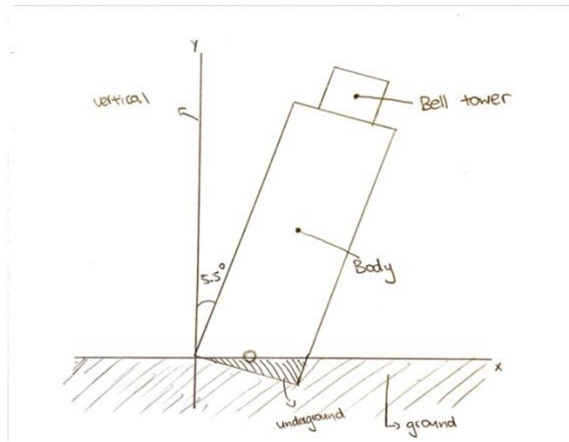

Figure 1. The general model of the leaning tower of Pisa

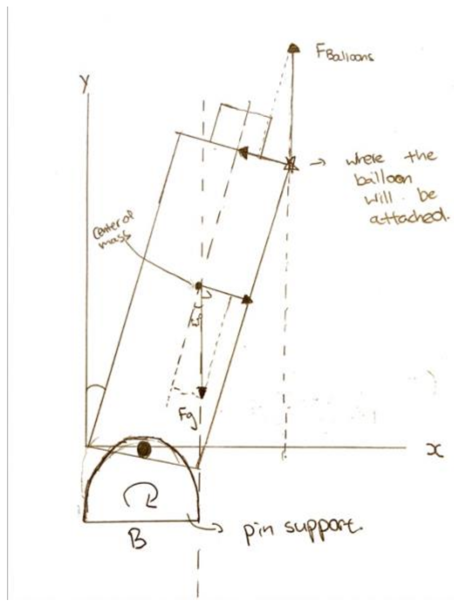

Figure 2. The general model modified with a imaginary pin support attached at the base of the tower

As can be seen in figure 2, the center of gravity and where the balloons will be attached to are also clearly labelled. These variables will be discussed next. For now, the model has simplified the situation to an equilibrium problem similar to ones encountered in Mechanics. In other words, the component force caused by gravity perpendicular to the tower's body will create a moment at point B, and our mission is to use balloons to create an upwards force that would create a counteracting component force which would make the resultant moment at B equal to zero. This would be done and calculated by using the equation [$(M_R)_B = \sum Fd$] modified from a similar equation in [3].

Now in order to continue, the different variables have to be found and defined through estimation. The center of gravity (Fg) is located somewhere roughly on the fourth floor of the tower [4] and is estimated to be 30m away from the base of the tower as the whole tower's original height is 60m [5] with a total of eight floors [6]. The component force created by balloons ($Fb_{component}$) is acting a distance of roughly 52m away from the base with opposite direction from that of the $Fg_{component}$ (can be seen in figure 3 where the tower of Pisa is being treated like a beam).

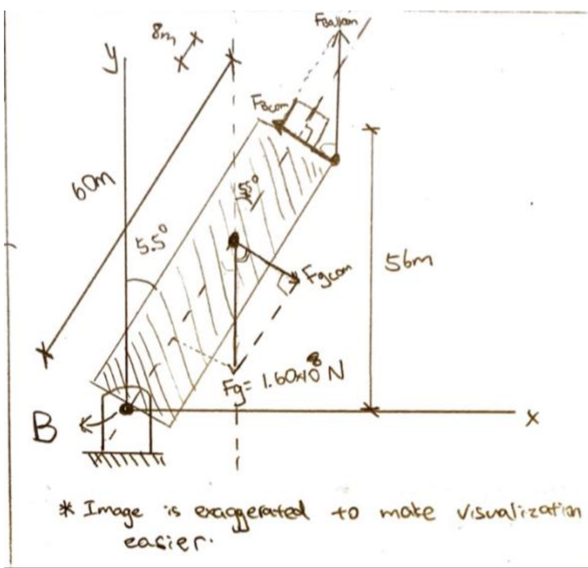

Figure 3. The previous model has ben simplified a bit more with components added in to treat the problem as an equilibirum problem in 2D.

Now, to find Fg, $Fg_{component}$, Fb, and $Fb_{component}$, the mass of the tower must be estimated first. It will be found in this case by estimating the tower's volume of construction blocks and multiplying that by the density of the material used (mostly built with Marbles). Before estimating, we must remember that the tower is actually hallow on the inside [6]. For simplicity, the tower will be thought of as a cylinder with its inner cylinder being carved out due to it being empty on the inside. Using fermi estimation and dimensional analysis, I was able to find a rough upper bound and lower bound (shown in table 1) and find their average mass of 1.64E7 kg (weight of 1.60E8N). In this table, the percentage of empty volume is an estimation based on the appearance of the tower on the outside (a total guess estimation not based on evidence).

Table 1. showing the values used for fermi approximation

	External Cylinder Radius	Internal Cylinder Radius	Height of cylinder (With bell tower and without)	Percentage of Empty volume (Guess)	Density of Marble ranges from 2600 to 2800 kg/m^3 [9]	Mass approximation	Average mass: 1.64E7 kg
Upper bound	16.9/2 meters obtained from [7]	7.66/2 meters obtained from [7]	60 meters	40 percent obtained from guessing	2600 kg/m^3	1.81E7 kg	
Lower Bound	16.9/2 meters obtained from [7]	7.66/2 meters obtained from [7]	52 meters obtained from using reference height of bell tower from [8]	40 percent obtained from guessing and looking at the exterior decorations of tower	2800 kg/m^3	1.46E7 kg	

By converting the tower's mass to weight, the force caused by gravity (Fg) is defined; following the free body diagram drawn in Figure 2, $Fg_{component}$ with a value of 1.60E8 N is found using simple trigonometry. Then by setting up the equation $[(M_R)_B= \sum Fd=$ $(-)Fg_{component}(30m)+Fb_{component}(52m)]$ and equating it to equal zero, $Fb_{component}$ is then found to be 8.83E6 N. Using simple trigonometry, the vertical force that balloons need to provide, Fb, is found to be 9.21E7 N.

Next, the balloons themselves need to be accounted for. In our problem, every single unit balloon is thought of as a perfect sphere having a standardized diameter of 11inches (0.28m) [10], since they are party balloons similar to the ones from the movie "Up." The volume of each balloon works out to be roughly 11 L. Establishing that 1 liter of Helium gas can generate roughly

0.0098 Newtons of force [11], we can obtain roughly how many balloons we need to generate 9.21E7 Newtons of upward force through dimensional analysis. Finally, that number turns out to be roughly 8.5E8 balloons (as sample calculations can be seen in Figure 4). However, the above estimation could only be relevant if several other perspectives are assumed. For our case, we are assuming that gravity doesn't weigh down on Helium itself and that the strings used to attach the balloons to the tower have no mass. Furthermore, the tower of Pisa must be a rigid body that won't break when exposed to compression or tension

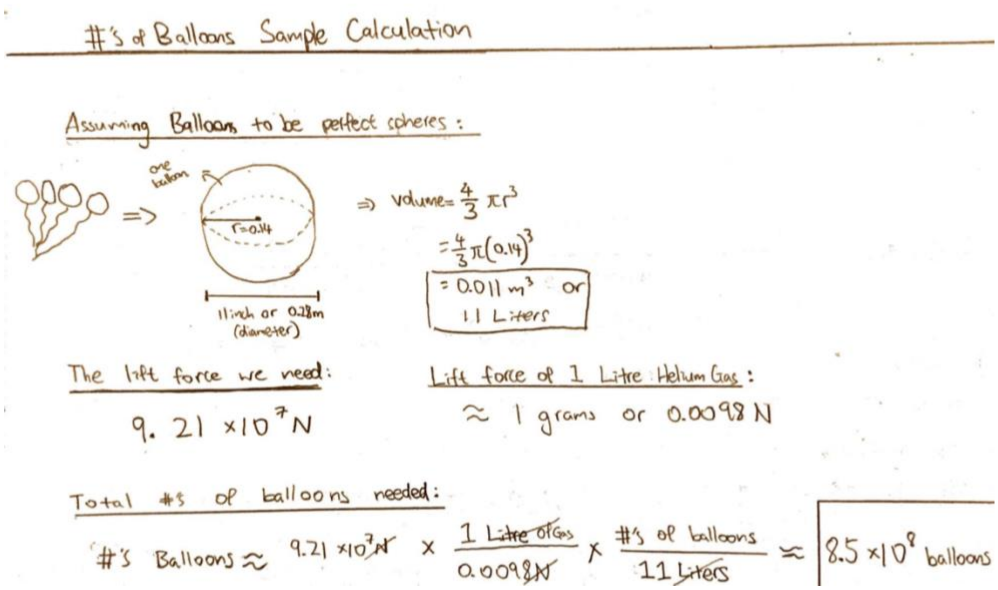

Figure 4 Numbers of Balloons and Sample Calculation

As a conclusion, this modelling project only attempted to solve part of the problem proposed. That is, ensuring that the tower of Pisa will not fall to its destruction through the establishment of equilibrium using balloons. To fix the tilt of the tower would increase in complexity as that would require more extensive knowledge and experience in solving problems using dynamics and understanding more about the rate of change of $Fb_{component}$ as the tower is slowly moved back to its original vertical.

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